Find the slope of the tangent line at the point $(-1,2)$ on the curve $x^4y^3 - x^3y = (x+y^2)^2 + 1$. **SCORE: _____ / 5 PTS**

$$(4x^3)y^3 + x^4\left(3y^2\frac{dy}{dx}\right) - \left[(3x^2)y + x^3\left(\frac{dy}{dx}\right)\right] = 2(x+y^2)\left(1+2y\frac{dy}{dx}\right)$$ ① ① ①

$$4(-1)^3(2)^3 + (-1)^4(3(2)^2)\left.\frac{dy}{dx}\right|_{(-1,2)} - \left[3(-1)^2(2) + (-1)^3\left.\frac{dy}{dx}\right|_{(-1,2)}\right]$$

$$= 2(-1+2^2)\left(1+2(2)\left.\frac{dy}{dx}\right|_{(-1,2)}\right)$$

$$-32 + 12\left.\frac{dy}{dx}\right|_{(-1,2)} - \left[6 - \left.\frac{dy}{dx}\right|_{(-1,2)}\right] = 6\left(1+4\left.\frac{dy}{dx}\right|_{(-1,2)}\right)$$ ①

$$-38 + 13\left.\frac{dy}{dx}\right|_{(-1,2)} = 6 + 24\left.\frac{dy}{dx}\right|_{(-1,2)}$$

$$-44 = 11\left.\frac{dy}{dx}\right|_{(-1,2)}$$

$$\left.\frac{dy}{dx}\right|_{(-1,2)} = -4$$ ①

Prove that the families of curves $y = \dfrac{a}{x^4}$ and $x^2 - 4y^2 = b$ are orthogonal trajectories. **SCORE: _____ / 5 PTS**

$$\frac{dy}{dx} = -4ax^{-5}$$ ($\frac{1}{2}$)

$$2x - 8y\frac{dy}{dx} = 0$$ ①

$$\frac{dy}{dx} = \frac{x}{4y}$$ ($\frac{1}{2}$)

$$-4ax^{-5}\cdot\frac{x}{4y} = \frac{-ax^{-4}}{y} = \frac{-y}{y} = -1$$

① ($\frac{1}{2}$) ① ($\frac{1}{2}$)

Find the following derivatives. Simplify all answers appropriately. **SCORE: _____ / 20 PTS**

[a] $\frac{d}{dt}\tan^{-1}\sqrt{t}$

$= \frac{1}{1+(\sqrt{t})^2} \cdot \frac{1}{2\sqrt{t}}$ 2 2

OK IF SIMPLIFIED $\rightarrow$

$= \frac{1}{2\sqrt{t}(1+t)}$

[b] $\frac{d}{d\theta}(\arcsin\theta)^{\cot\theta}$

$y = (\arcsin\theta)^{\cot\theta}$

1 $\ln y = \cot\theta \ln \arcsin\theta$

1 $\frac{1}{y}\frac{dy}{d\theta} = -\csc^2\theta \ln\arcsin\theta$ 1

2 $+ \cot\theta \frac{1}{\arcsin\theta} \frac{1}{\sqrt{1-\theta^2}}$

$\frac{dy}{d\theta} = y\left(\frac{\cot\theta}{\sqrt{1-\theta^2}\arcsin\theta} - \csc^2\theta\ln\arcsin\theta\right)$

2 $= (\arcsin\theta)^{\cot\theta} \cdot \left(\frac{\cot\theta}{\sqrt{1-\theta^2}\arcsin\theta} - \csc^2\theta\ln\arcsin\theta\right)$

[c] $\frac{d}{dy}\ln\frac{\sqrt[3]{4y^2-7}}{(4-7y^2)^5}$

$= \frac{d}{dy}\left[\frac{1}{3}\ln(4y^2-7) - 5\ln(4-7y^2)\right]$ 1

$= \frac{1}{3}\cdot\frac{1}{4y^2-7}\cdot 8y - 5\cdot\frac{1}{4-7y^2}\cdot -14y$

$= \boxed{\frac{8y}{3(4y^2-7)}} + \boxed{\frac{70y}{4-7y^2}}$

2 2

[d] $\frac{d}{dx}\sec(xe^{\cos x})$

OK IF SIMPLIFIED

$= \sec(xe^{\cos x})\tan(xe^{\cos x})\cdot$ 1

1 $(e^{\cos x} + xe^{\cos x}(-\sin x))$ 2

$= \sec(xe^{\cos x})\tan(xe^{\cos x})\cdot(e^{\cos x} - xe^{\cos x}\sin x)$

$= e^{\cos x}\sec(xe^{\cos x})\tan(xe^{\cos x})\cdot(1 - x\sin x)$